ELLIPTICAL COLLINEAR RESTRICTED FOUR-BODY PROBLEM

SHORT SYNOPSIS

Submitted in partial fulfillment of the requirement of the degree of

DOCTOR OF PHILOSOPHY

by

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Abstract

The elliptic R3BP research studies how a small object moves when it is influenced by the gravitational pull of three larger bodies that are moving in elliptical orbits. This synopsis is about to examine the elliptic restricted four-body problem, which consisting of both cases: a radiating central dominant mass and a non-radiating dominant mass. The problem in the circular case has been already developed in a rotating coordinate system, which includes mass-ratio, radiation pressure effects, oblateness effects, etc.

This research will determine the equilibrium (libration) points and evaluate their dependence on critical system parameters like eccentricity, radiation factor, and mass ratio are the primary objectives of this study. To determine how variations in these parameters influence the position and existence of equilibrium points, a systematic parametric analysis will be carried out. Additionally, linear stability theory (Lyapnouv Stability) will be utilized to analyze the nature and stability of these points, which will provide the clarity on their dynamic behaviour. Further, we will explore the possible regions of motion for the space vehicle in the proposed dynamical system.

The findings will illustrate that the positions of equilibrium points and the system's overall dynamical structure are significantly influenced by both radiation pressure and orbital eccentricity. The results will improve the understanding of multi-body gravitational interactions and may be valuable in mission planning, specifically when it will involve in designing spacecraft trajectories and stability regions in complex gravitational environments.

Keywords: Elliptic Restricted Four-Body Problem, Collinear Equilibrium Points, Stability Analysis, Radiation Pressure, Orbital Eccentricity.

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List of Abbreviations

R3BP	Restricted Three-Body Problem
ER3BP	Elliptic Restricted Three-Body Problem
R4BP	Restricted Four-Body Problem
CR4BP	Collinear Restricted Four-Body Problem
ECR4BP	Elliptical Collinear Restricted Four-Body Problem
EPs	Equilibrium Points
LPs	Libration Points
CoM	Centre of Mass

1. Introduction

1.1 R3BP

The classical R3BP arises when the two massive bodies are orbiting in a circular motion around their mutual CoM, and the third body, having infinitesimal/negligible mass, is moving in the same orbital plane defined by the massive bodies. Because of their mutually strong gravitational attraction, the two primaries in this problem possess finite masses and move in perfectly circular paths, on the other hand, small body has very small mass and does not influence the primaries' motion.

To evaluate the motion of infinitesimal mass is called the R3BP. To simplify the analysis, the system is usually defined in a rotating coordinate frame in which the positions of the two primaries can be seen as constants because the line that connects them stays stable. A set of nonlinear differential equations evaluate the motion of the infinitesimal particle in this rotating frame. These equations consider the gravitational forces of the massive bodies as well as the centrifugal and Coriolis forces that come from rotation. You can use an effective potential function that considers and takes in both rotational and gravitational effects to describe how the system operates. The configuration of this model is given in Fig.1, where m_1 and m_2 are the most and least massive bodies, respectively. The infinitesimal mass is denoted by m_3; r_1, r_2 and r are the respective distances of the infinitesimal mass from the primaries m_1, m_2 and CoM respectively.

These assumptions hold for the model presented:

- It is assumed that the distance between the two massive bodies is one.
- It is assumed that the sum of the masses of the massive bodies is one.
- The time unit is chosen such that the value of G becomes one.
- During the entire process, the major axis of the massive bodies remains parallel to the synodic axes.
- The orbital plane must be include the equatorial plane of the massive bodies.

The equation of motion of the infinitesimal mass in R3BP are given by (Szebehely, 1967):

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \Omega_x \\ \ddot{y} + 2n\dot{x} &= \Omega_y \end{aligned}\right\} \tag{1}$$

where, $\Omega(x, y)$ is the effective potential function, define as:

$$\Omega = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}. \tag{2}$$

Here,

μ is the mass ratio, defined as: $\mu = \frac{m_2}{m_1 + m_2}$,

r_1 is the distance of infinitesimal mass from the smaller primary,

r_2 is the distance of infinitesimal mass from the larger primary,

(x, y) are the co-ordinates of infinitesimal mass.

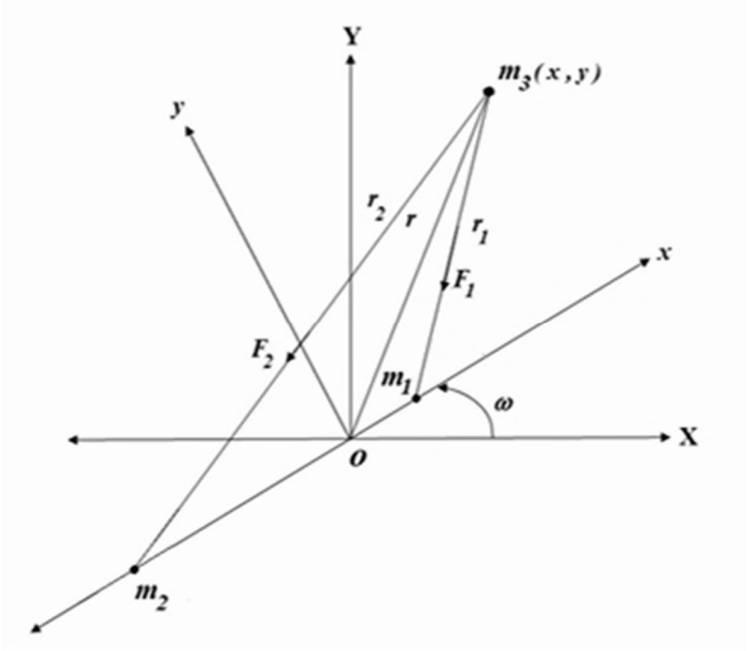

Fig. 1: Configuration of the R3BP

The R3BP possesses of five equilibrium points (Libration points); three of which are collinear and two are non-collinear. The collinear equilibria are unstable, on the other hand the non-collinear equilibria are stable for a critical mass parameter μ_c = 0.0385209… (Szebehely, 1967). This model is used to explore EPs and their stability in the real planetary systems, e.g., the Sun-Earth system and the Earth-Moon system.

1.2 Collinear Restricted Four-Body Problem

The CR4BP is a particular case of the classical R4BP in celestial mechanics, where three massive bodies (primaries) travel under their mutual gravitational attraction, and a fourth body of infinitesimal mass moves under their gravitational influence without affecting their motion. This model helps us to study the dynamics of minor planets (asteroids, planetoids, comets, etc.) under the gravitational field of three massive bodies (Michealodimitrakis, 1981).

In the *collinear configuration*, all three primaries lie along a straight line at all times, Fig. 2.

The CR4BP is formulated under the following assumptions:

- The three primaries, with masses m_1, m_2, m_3 are arranged collinearly on a straight line.
- The motion of the primaries is determined by Newtonian gravitational law, and they form a known solution of the three-body problem (typically a rotating configuration).
- The fourth body has negligible mass and does not affect the motion of the primaries.
- The system is analyzed in a rotating reference frame where the primaries remain fixed on the x-axis.
- Units are normalized such that:
    - Total mass $m_1 + m_2 + m_3 = 1$.
    - Distance between primaries is unity.
    - Gravitational constant $G = 1$.

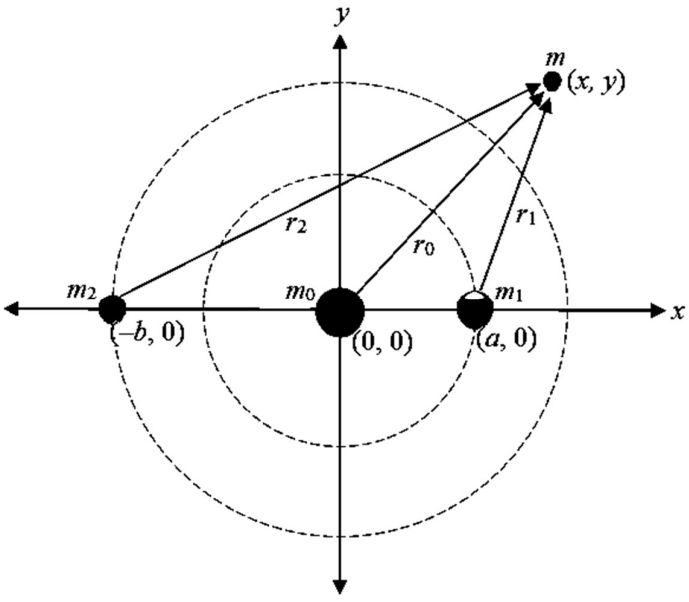

Fig. 2: The configuration of collinear concentric restricted four-body problem

Equations of Motion

Let the positions of the massive bodies be fixed at $x = x_1, x_2, x_3$ on the x-axis in a rotating coordinate system. Let (x, y) denote the coordinates of the infinitesimal particle.

The equations of motion are given by Idrisi et al. (2025):

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \Theta_x \\ \ddot{y} + 2n\dot{x} &= \Theta_y \end{aligned}\right\} \tag{3}$$

The potential function $\Theta(x, y)$ can be expressed as:

$$\Theta(x,y) = \frac{n^2}{2}(x^2 + y^2) + \frac{\mu_0}{r_0} + \frac{1-\mu-\mu_0}{r_1} + \frac{\mu}{r_2}. \tag{4}$$

Here n is the mean-motion of the primaries, and

$$\mu_0 = \frac{m_0}{M}, \mu = \frac{m_2}{M}, M = \sum_{i=0}^{2} m_i = 1,$$

$$r_0^2 = x^2 + y^2,\ r_1^2 = (x-a)^2 + y^2,\ r_2^2 = (x+b)^2 + y^2,$$

$$a = \frac{\mu}{1-\mu_0}, b = \frac{1-\mu-\mu_0}{1-\mu_0}; 0 < \mu < \frac{1-\mu_0}{2} \text{ and } \frac{1}{2} < \mu_0 < 1.$$

The system admits a conserved quantity known as the *Jacobi integral*, given by:

$$C = 2\Theta(x,y) - (\dot{x}^2 + \dot{y}^2) \tag{5}$$

This integral plays a crucial role in determining the allowable regions of motion (zero-velocity curves).

Equilibrium Points (EPs) / Libration Points (LPs)

The equilibrium points are obtained by solving:

$$\frac{\partial \Theta}{\partial x} = 0, \frac{\partial \Theta}{\partial y} = 0.$$

For the collinear configuration:

- The equilibrium points lie on the x-axis ($y = 0$), are called *collinear equilibrium points*.
- The equilibrium points lie on the xy-plane ($x \neq$ and $y \neq 0$), are called *non-collinear equilibrium points*.
- The number of equilibrium points depends on the mass distribution and configuration of the primaries.
- Typically, more equilibrium points exist compared to the classical restricted three-body problem.

Stability Analysis

To analyze stability, small perturbations are introduced around equilibrium points:

$$x = x_0 + \xi,\ y = y_0 + \eta$$

Linearizing the equations of motion leads to a system of variational equations:

$$\left.\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= \xi \overset{o}{\Theta}_{xx} + \eta \overset{o}{\Theta}_{xy} \\ \ddot{\eta} + 2n\dot{\xi} &= \xi \overset{o}{\Theta}_{yx} + \eta \overset{o}{\Theta}_{yy} \end{aligned}\right\} \qquad (6)$$

where 'o' indicates that the partial derivatives are to be calculated at the libration points under consideration.

The characteristic equation that relates to the system of Eqns. (12), we get

$$\lambda^4 + \left(4n^2 - \overset{0}{\Theta}_{xx}(x_0, y_0) - \overset{0}{\Theta}_{yy}(x_0, y_0)\right)\lambda^2 + \overset{0}{\Theta}_{xx}(x_0, y_0)\overset{0}{\Theta}_{yy}(x_0, y_0) - \left(\overset{0}{\Theta}_{xy}(x_0, y_0)\right)^2 = 0. \qquad (7)$$

The characteristic Eqn. (3) derived from this system determines the nature of eigenvalues:

- **Pure imaginary eigenvalues** → Stable motion
- **Real eigenvalues** → Instability
- **Complex with real part** → Saddle-type instability

In most cases, the collinear points are unstable on the other hand non-collinear are stable under some conditions.

Applications

The CR4BP has several important applications in modern astrophysics and space dynamics:

- Motion of spacecraft influenced by multiple celestial bodies (e.g., Sun–Earth–Moon systems)
- Study of asteroid dynamics in multi-body environments
- Stability analysis of dust particles in planetary rings
- Modeling complex gravitational environments in exoplanetary systems.

Special Cases

- If the central mass tends to zero, the system reduces to the *CR3BP*.
- Symmetric mass configurations simplify the analysis and may lead to analytical approximations of equilibrium points.

The *Collinear Restricted Four-Body Problem* extends the classical three-body framework by incorporating an additional primary, leading to richer dynamical behavior and more complex equilibrium structures. Its study provides deeper insights into multi-body gravitational systems and has significant relevance in both theoretical and applied celestial mechanics.

Elliptical Collinear Restricted Four-Body Problem (ECR4BP)

The elliptical collinear restricted four-body problem can be applied in studying the dynamics of the infinitesimal mass. However, this model could be useful since planets revolve in elliptical orbits and not circular orbits. This research paper focuses on modelling the system to formulate the existance and stability of equilibrium points, considering the radiation and orbital eccentricity of the primaries. The ECR4BP model is regarded as among the most important approaches in celestial mechanics where the motion of the infinitesimal mass subjected to gravitational attraction of the other three masses in a straight line are studied. In this case, the central dominant mass provides the gravitational field and is assumed to be relatively very massive and fixed at the centre of mass point. Among the other two primaries, one primary is more massive than the other and orbiting around the central dominant mass in elliptical paths. As a result of the small size of the body, the impact of the gravitational force of the primaries will not influence the movements of the primaries. Furthermore, considering the elliptical

movement of the primaries, the movement becomes a function of time. Mathematical modelling becomes extremely handy in situations like these in terms of equilibrium point stability. The proposed problem is practically relevant because the planetary movement is not circular but elliptical. Our objective in this research is to construct a model and analyse the EPs and their stability regarding radiation and eccentricity effects of the larger particles.

2. Literature review

The circulation of objects in the space was one of the major themes of celestial mechanics as far back as the Newton law of universal gravitation was invented. This can happen due to gravitational attraction between two objects. However, when a third object is introduced into the scene, the system becomes highly non-linear, and an analytical solution cannot be achieved. The first person to explore the existence of EPs in this system was Joseph-Louis Lagrange (1772), who suggested the concept of five EPs. The knowledge gained about EPs is very helpful for trajectory planning and mission design. A mathematical theory, which includes R3BP, Equilibrium Points, periodic orbits, and their solutions, was developed by Szebehely (1967). Regarding periodic orbits in the Earth-Moon system. Broucke (1968) investigated the R3BP. Generalisation of the problem to the restricted four-body problem can be made where the four body (massless and moving at negligible speed) revolves around specific orbits due to gravitational forces acting upon it from the other three bodies. This study was done by Michalodimitrakis (1981) in the field of motion of an infinitesimal body due to attraction by the other three bodies.

These were significant works of Poincaré (1993) to the qualitative study of the three-body problem because he demonstrated that the general three-body problem was unsolvable in closed form and gave contemporary methods of dynamical systems to examine gravitational systems. Murray and Dermott (1999) contributed to a paper about the challenging restricted three-body problem that examines at how planets in a planetary system affect their mutual gravitational pull. Roy (2005) analyzed the orbital dynamics of planets, highlighting the gravitational interactions among the planets inside a particular astronomical system; this concept was subjected to examination and study. Baltagiannis and Papadakis (2011) investigated into the stability and equilibrium points of a four-body problem with restrictions within a circle. They discovered that the four-body problem requires a larger number of equilibrium points compared to the restricted three-body problem.

ER3BP was discussed in Singh and Umar (2012), where it was assumed that the large primary body radiates while the smaller primary body is an oblate one. It has been observed that there exist orbits that are stable concerning the EPs; however, these orbits become fewer due to the influence of radiation and oblateness. Kumari and Kushvah (2014) have considered the effect of oblate primaries on regions of stability of EPs. The role of oblateness on stability of EPs has

been studied here. In Falaye (2015), the effect of radiation and oblateness on the motion of an infinitesimal mass is studied.

Chakraborty et al. (2016) examined the presence and stability of collinear equilibrium points in ER3BP and showed how variables like radiation pressure and oblateness can change the system's dynamical structure.

The ER4BP is a more practical variation of the R4BP in which the primaries travel in elliptic orbits rather than circular ones. Since a fully circular motion is rarely seen in real astrophysical systems, this model is very helpful for explaining them. Chakraborty and Narayan (2019) examined a constrained R4BP in which three massive masses move in an elliptic, triangular manner. The study focused on how equilibrium point locations are impacted by the eccentricity of these orbits . Dewangan et al. (2020) examined the stability of EPs in the generalized ER4BP and shown that the position and stability of LPs are strongly influenced by radiation pressure and oblateness. Younis et al. (2021) investigated how radiation pressure and the elliptic paths of the planets move a small object about in a particular case of the ER4BP applied to our own Earth-Moon-Sun system. Carletta et al. (2022) examined low-energy and quasi-periodic trajectories in the ER4BP and showed how ER4BP might be implemented in mission planning and spacecraft trajectory design.

In a more recent study, the ER4BP was examined by Dewangan et al. (2023) considering the effects of radiation pressure and primary oblateness. They discovered that the number of EPs depends on the true anomaly of the orbit after analysing the position and stability of collinear and non-collinear Libration points. Additionally, their findings demonstrated the linear instability of both planar and non-planar equilibrium sites. Alrebdi et al. (2024) studied the collinear restricted four-body problem by taking into consideration the fundamental bodies' oblate and prolate non-spherical forms. Meena and Kishor (2024) explored the effects on a small object of radiation from two stars and the elliptic orbits of the masses. It demonstrated how the magnitude of the masses and the "push" from radiation drastically alter the timing and form of orbits. Suraj et al. (2024) studied the impact of oblateness and prolateness on the position and stability of EPs in the CR4BP with a non-spherical central primary. They demonstrated how system characteristics affect the type and quantity of equilibrium points.

Muscas (2025) explored the EPs and their stability in the CR4BP with logarithmic potential. Additionally, they demonstrated that several dynamical features apply to the non-restricted situation. Zhao et al. (2025) investigates the CR4BP, in which three primary orbits around their

shared CoM in circular paths and are positioned at the vertices of an equilateral triangle. They believed that the motion of the massive bodies is unaffected by the fourth body, which has a very negligible mass. They demonstrated the existence of non-collision periodic solutions for both equal and unequal mass distributions using variational techniques. A generalised framework for the collinear restricted four-body problem considering a central dominant mass was recently established by Idrisi et al. (2025). Ma et al. (2026) examined the collinear restricted four-body problem while considering perturbations such fluctuating mass, radiation pressure, oblateness, and Stokes drag. They examined the stability and the existence of EPs and associated attraction basins, demonstrating that while Stokes drag eliminates collinear equilibria and has a significant effect on the basin structure, the count of EPs changes with system parameters.

The dynamical structure related to the sysytem when one of the primary functions acts as a dominant central body was examined by the authors. They investigated how motion of the tiny body is impacted by the gravitational influence of the central mass and concentrated on the existence and the behaviour of collinear equilibrium configurations. The findings demonstrated that the equilibrium structure and dynamical properties related to the system are considerably altered by the existence of a dominant central mass. The paper helps in improving the exploration of the various complex forms of the R4BP and provides a theoretical basis to study real-world examples of multi-body celestial objects. Almost all the current efforts have focused on either the circular R4BP or dynamical configurations. However, this is contrary to the existing literature on the success gained from the exploration of the restricted four-body problem, especially the identification of the EPs and their stability. Very little work has been done on the elliptic restricted four-body problem, where the massive bodies move in elliptical motion. In addition, studies involving collinear EPs of the ER4BP have not been widely investigated, especially the influence of the variation in the pressure of radiation. Further, consideration has been given to the ignorance by one of the dominant masses about the force of radiation in comparison to the earlier analysis of the ER4BP where the study revealed that both the dominant masses radiated equally.

It was analyzed that there had not been any concern about Radiation and Non-Radiation dominating mass during the exploration of the ER4BP. In fact, there is minimal or no information about the way such condition(s) can influence the location of the EPs. Thus, the exploration of the ER4BP with one or more radiating and one or more non-radiating dominant

mass at once is highly required to enhance our knowledge on multi-body gravitational systems and their usage.

3. Research Gap

Although the three body problem and four body problem both of celestial mechanics are extensively examined, in most cases analyzed are simplified problems in which the major planets are restricted to a circular orbit, and radiation pressure is not considered.

Based on the literature review, the literature gaps regarding the R4BP present are the following:

- Most of the research work conducted on R4BP is with circular orbit as compared to a more realistic situation which involves elliptical orbit.
- Lessons learned: Scarcity of detailed examination of collinear structures in the ER4BP in the extant literature has been the case.
- ER4BP did not get as much attention as CR4BP.
- The interplay between orbital eccentricity and the EPs and their stability have not been considered appropriately.
- There is no comprehensive study on the dynamical properties of the system to radiating and non-radiating conditions of a central dominant mass.

4. Problem Statement

To derive and analyse a perturbed dynamical model of the elliptical collinear restricted four-body problem, which takes the impact of orbital eccentricity and the impact of radiation pressure into account.

5. Objectives of Research

The following are the goals of the proposed study:

1. To find the equations of motion of the elliptic collinear restricted four-body problem:

    A. in the case that there is no central dominant mass radiation,

    B. in the case when the central dominant mass is a radiating body.

2. To establish the analysis for the presence and specific location of equilibrium (Libration) points when:

    A. the dominant primary is non- radiating,

    B. the dominant primary is radiation one.

3. To determine how the

    A. eccentricity and

    B. radiation pressure

influence the linear stability of the EPs.

4. To make a comparison of the dynamical behaviour of the system under radiation and elliptic cases with the existing literature.

6. Significance of the Study

The proposed study will make a major contribution to the exploration of celestial mechanics and dynamical systems with the following aspects:

- The classical R4BP is extended by the elliptic motion of primaries, which is a more realistic description of most astrophysical systems.
- This research will improve the understanding of the dynamical nature of infinitesimal bodies in multi-body gravitational systems.
- The radiation pressure effect can be added to the model to enable more specialised applications to radiating bodies.
- The analysis of equilibrium positions will help us to discuss the behaviour of small particles, spacecrafts and cosmic dust in an external gravity field.
- The concept of elliptical restricted multi-body dynamical systems, which are considered to be relatively new and have not been developed as compared to the circular restricted multi-body system, will be developed using the research.
- Designing the space vehicle trajectories and studies of space vehicle gravitational stability will also be useful in terms of the survey.

7. Methodology

The theory of celestial mechanics and the elliptic restricted four-body problem are some of the areas that are meant to be studied by this research. The various steps followed by a natural progression in the approach to addressing a research problem direct the research methodology. The paper begins with a literature review, and the research work aims at the R4BP, collinear configurations, and radiation pressure. This will be subsequently, a model constructed that will be on the basis of the assumptions that two of the three bodies will travel in elliptical orbits, where one of them has a larger mass and will be stationary at the CoM of the system.

Formulation of Mathematical Model: We shall construct the mathematical model of the proposed problem in the same way as that described by Idirisi et al. (2025).

Analysis of the existence of Equilibrium Points: The existence criteria of all EPs (collinear and non-collinear both) will be analyzed in this study under the consideration of orbital eccentricity and radiation factor.

Stability of EPs: The linear stability analysis of the equilibrium points, collinear as well as non-collinear, will be conducted. The emphasis of the current investigation will be laid on the impact of radiation and eccentricity on the presence and stability of the EPs.

Numerical Simulation: The software package MATHEMATICA will be utilised for simulation purposes. It will turn out to be helpful in illustrating the effects of radiation and eccentricity on the EPs.

Validation: A process of validation will be undertaken to ensure that the results arrived at are sound by comparing them with known results or comparisons of past research.

Interpretation and Conclusion: There will be an interpretation and a conclusion of the results obtained. Physical significance of the results will be analysed, and the system structure of the results. Additionally, the radiation stress concerning the dynamical structure system shall be addressed.

8. Expected Outcomes of Research

The outcomes that the proposed study is set to generate are the following:

- Investigating how radiation pressure and orbital eccentricity affect the equilibrium points' position and dynamic behaviour.
- Development of the elliptic collinear restricted four-body problems: Mathematical formulation and equations of motion, including radiating and non-radiating dominant mass.
- To ascertain the circumstances of stability or instability, the linear stability of the EPs under various system parameters is analysed.
- Determining whether the collinear equilibrium (Libration) points in the system's infinitesimal mass exist and are stable.
- Numerical computations and graphic depictions are used to validate the analytical conclusions, offering insights into the system's dynamical structure and its significance for astrodynamics and celestial mechanics.

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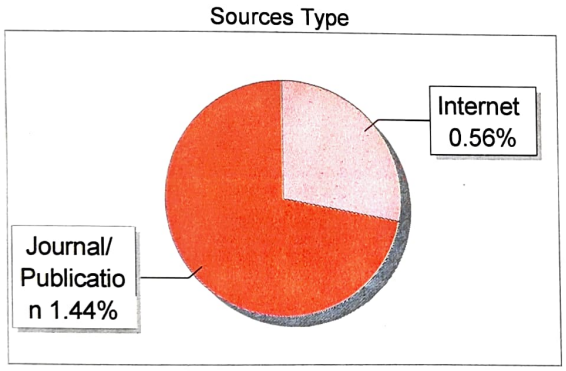

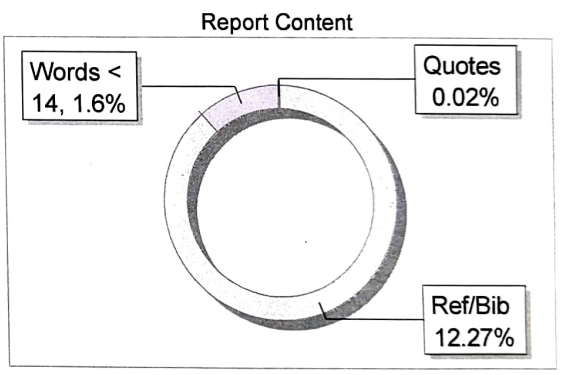

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Elliptical Collinear Restricted Four-Body Problem

SHORT SYNOPSIS

by

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Abstract

The elliptic R3BP research studies how a small object moves when it is influenced by the gravitational pull of three larger bodies that are moving in elliptical orbits. This synopsis is about to examine the elliptic restricted four-body problem, which consisting of both cases: a radiating central dominant mass and a non-radiating dominant mass. The problem in the circular case has been already developed in a rotating coordinate system, which includes mass- ratio, radiation pressure effects, oblateness effects, etc.

This research will determine the equilibrium (libration) points and evaluate their dependence on critical system parameters like eccentricity, radiation factor, and mass ratio are the primary objectives of this study. To determine how variations in these parameters influence the position and existence of equilibrium points, a systematic parametric analysis will be carried out. Additionally, linear stability theory (Lyapnouv Stability) will be utilized to analyze the nature and stability of these points, which will provide the clarity on their dynamic behaviour. Further, we will explore the possible regions of motion for the space vehicle in the proposed dynamical system.

The findings will illustrate that the positions of equilibrium points and the system's overall dynamical structure are significantly influenced by both radiation pressure and orbital eccentricity. The results will improve the understanding of multi-body gravitational interactions and may be valuable in mission planning, specifically when it will involve in designing spacecraft trajectories and stability regions in complex gravitational environments.

Keywords: Elliptic Restricted Four-Body Problem, Collinear Equilibrium Points, Stability Analysis, Radiation Pressure, Orbital Eccentricity.

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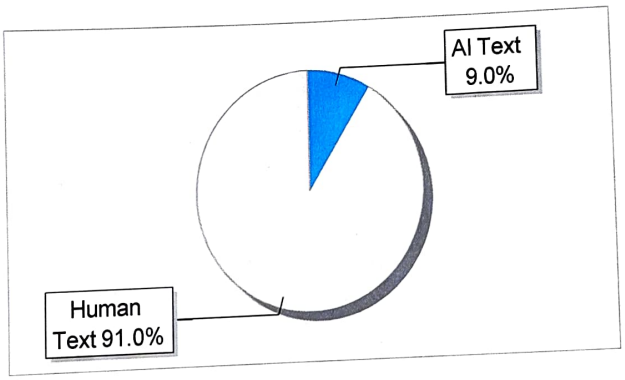

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